Brain research and mathematics education: some comments

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Abstract

Combining brain research and mathematics education can be advantageous but it is important to be aware of the risks. The present comments focus on Damasio (2010) "Self comes to mind. Constructing the conscious brain" and Colignatus (2011) "Conquest of the plane", a primer for the education of mathematics. Mathematical concepts like line and circle are abstract notions like the other mental events that make up the mind. Education can use this to its advantage. Van Hiele levels of abstraction can guide this research. Freudenthal's "realistic mathematics education" tends to contain too much distraction from the required abstract development, and is not the proper answer to the current problems in mathematics education. The main problem in mathematics education is that mathematicians are trained for theory while when they meet students in class then these are real life students, which requires an empirical mind set. Current mathematics education tries to solve its problems by resorting to tradition but this causes mathematical concepts and notations that can be cumbersome and actually hinder understanding. Such confusions need to be resolved, otherwise the link up of brain research and mathematics education will have no results, or even negative ones. The discussion of brain research applied to abstract notions such as line and circle requires a firm foundation in the philosophy of science, and a perception on the Platonic world frequently favoured by mathematicians.

Contents

Introduction

Mathematics education can benefit from brain research in various ways. The literature however contains some confusing concepts. Confusion comes with the risk that wrong concepts would become enshrined in education. The intention of the following comments is to generate clarity.

Damasio (2010) defines the mind as consisting of mental events, both conscious and nonconscious (rather than unconscious). If the image of a circle in the mind, e.g. as the Platonic object, is to be a mental event, then supposedly it might exist also nonconsciously. But thinking about a circle is different from not thinking about it. It seems more likely that the circle comes into being as a mental event because of the thought. Damasio also clarifies that the whole brain is required to create consciousness. If the image of the circle exists only because it is created in the conscious mind, then there would be no similar image in the nonconsciousness since this lacks the whole supporting apparatus. If this holds for all such mental events, does that not imply that nonconsciousness does not use *quite* the same mental events as consciousness ? The model then rather is that mind = consciousness (with the equality sign), while nonconscious events are issues for the brain only and have no mental counterpart but only a supporting role. In other words, the circle exists only in the mind = consciousness while the brain keeps components that can be collected by consciousness to form that circle.

The other source of confusion is, remarkably, mathematics itself.

One example is this. My book *A Logic of Exceptions* (1981, 2007, 2011) (ALOE) shows that three-valued logic is required to deal with the logical paradoxes like the liar paradox or Russell's paradox, including Gödel's "incompleteness theorems" (i.e. rather confusions). If brain researchers follow current mainstream mathematical thought and are stuck in two-valued logic (like the authors of *Logicomix*), they might miss out on some phenomena in the brain related to three-valuedness. Their findings might be less relevant for good education.. (Though they would fit mainstream teaching, and potentially, if two-valuedness is the norm, those brain parts might be 'cured' that support three-valuedness.)

Another example is this. In mathematics education, Pierre van Hiele created the theory of levels. The same words can relate to different concepts, depending upon the level of understanding by the student. We know from Wittgenstein of the *Philosophical Investigations* with his language games that words are flexible anyway. Still, in mathematics and its education there remains a tendency to think that definitions and proofs can be formulated exactly. If one mathematician formulates the Pythagorean Theorem then another shows his understanding by proving it. If brain research is to try to support education in this, how can it deal with the gap between Wittgenstein's language games and the mathematical claim of exactness ? Pierre van Hiele's levels provide a bridge. But if the mathematical symbolism is formulated crookedly, for example with mixed numbers like 2½ that should rather be conceptualized and also written as 2 + ½, then we need a bridge *plus* a supporting coil and perhaps an additional twist. The circus on fractions in elementary school derives from failed didactics, and this would hinder brain research rather than create scope for its practical use.

As mathematics and its education are best developed within philosophical understanding, the discussion below will also consider Cartesius's "Cogito ergo sum" and consciousness and the self. Mathematics is often understood in Platonic abstraction, and its elements might well be as abstract as the mind itself. A circle as an ideal object might be made of the same 'stuff' as the mind itself.

My books *Elegance with Substance* (2009) (EWS) and *Conquest of the Plane* (2011) (COTP) have short references to brain research, notably Damasio *Looking for Spinoza* (2003) (LFS). It will be useful to extend a bit and also update to Damasio *Self comes to mind* (2010) (SCTM). Perhaps the title should rather read "Some comments on Damasio and EWS & COTP" but the impression is that there is a wider consequence.

Prolegomena

When we combine philosophical ideas with practical matters such as brain research or didactics of mathematics then all kinds of angles creep in that increase confusion. It will be useful to mention some foundational points without further discussion.

(1) ALOE p179 discusses that there are three kinds of approaches – determinism, randomness and volition – but that there is no experiment to determine which is the true state. At best we can design models that illuminate the different angles and act with pragmatic choice.

(2) Feeling and rationality can be seen as different dimensions rather than as opposites to one another (in only one dimension). (Colignatus (1996) figure 2.) Though they may occur in the mind/brain as a soup, and be linked by necessity, they can be separated by abstract analysis.

(3) One way to see the brain is as a processor of information. Feelings can be seen as information at emotional locations. The idea is to handle information as efficiently as possible, at lowest cost, in particular of energy. A cheap way to handle (conflicting) information is to discard it (cognitive dissonance). An efficient manner is to create definitions with maximal explanatory power. The notion 'all swans are white' can be treated as a definition rather than an empirical proposition that can be contradicted by a black swan.

(4) Morality is related to survival. David Hume correctly diagnosed that 'is' and 'ought' are different dimensions. This distinction however is analytical while the powerful impact of morality derives from the need for survival (of the species). A bridge between 'is' and 'ought' might be to behave as is predicted what will be morally best.

Our concept of reality

When clarifying the terminology for reality, a useful aspect is the concept of *continuity*. An example is a circle. As neurons are discontinuous, how does the brain handle 'reality' ?

Basic reality can be denoted as **reality0** and we will generally not know it. An example is an electron, which is a theoretical term. Kant used the term 'noumenon' for such things-in-themselves that we will never fully know. That term however is somewhat problematic since we may still know some things, such as electrical charge (also a theoretical term) of that electron. It suffices to assume reality0 without getting lost in questions about what 'can' be known about it.

Then there is the **reality1** created by the map in the brain. Apparently there are layers of cells, and the person who looks at a circle that is drawn on paper or thinks about a circle apparently has nerve cells activated indeed in such a circular form. Clearly, though, those active cells are in discrete number, and the notion of continuity must be added separately.

Subsequently, there is **reality2,** for example a circle in the mind. Reality2 differs from reality0 and perhaps does not deserve the label 'reality'. Yet it functions in the mind as our reality and thus may be labelled so. Obviously there are versions reality2a, reality2b, … with different flavours (language games).

Note that reality1 is much like reality0, in that we can hardly verify how the brain actually maps reality0. Thus reality1 would rather tend to be a model that we make in reality2. But it can be idenfied as a logical phase.

Within reality2 there are space2 and time2. It is not entirely sure that there are similar aspects within reality0 that we could call space0 and time0. Most likely it is rather useless to query about space0 and time0. It suffices to identify reality0 as a container. It could be sufficient that concepts space2 and time2 work.

Space2 is the concept of space as it naturally arises in the mind, and it is Euclidean. Within that space2 we can imagine other models, for example the non-Euclidean rules for the surface of a sphere. It is dubious whether it is wise to call these models 'space' too. A name is only a name, but if there are ontological suggestions then we should be rather careful about those.

My impression is that physicists are hopelessly confused in their verbal explanations about their models of reality. When two objects pass each other in free space then there is no obvious frame of reference, which is the base for the theory of relativity. But if I turn my head then it is silly to take my head as the frame of reference and say that the universe is spinning around – since huge masses would cover huge distances in mere seconds. It makes sense to look for the center of gravity in the universe for a frame of reference. In the 19th century physicists lost much time in discussing the 'aether' and they resolved their confusions by concentrating on measurement results. Einstein's 'contracting space' describes such measurements, by resolving all measurement errors into a 'flexible space'. On the contrary, I think that it is better to remain in Euclidean space and continue speaking about measurement errors. Otherwise the notion of 'space' is badly defined. I myself can only understand non-Euclidean 'space' like the surface of a sphere when I imagine that sphere within an Euclidean space. A short discussion is in COTP chapter 14.

Consciousness as a primitive concept

Consciousness is a primitive concept. It is understood by the conscious person. If it is not experienced then it cannot be explained. If it is experienced then its existence can be communicated to another person who experiences consciousness as well; but only its existence, and it still cannot be explained what it is.

The brain creates the mind apparently in electrical pulses of a certain length. The eye only has a small sensitive focus and it scans the world to build an image, that fits within those pulses. If awareness indeed has a full delay of 0.2 milliseconds then an atomic blast that destroys the brain leaves a mental after-life of 0.2 milliseconds. Likely though there still is some 'real-time' linkage.

Looking for a simple model of this kind I found Bruce Hoeneisen (2002) at Fermi Lab. The short paper and model explain how feedback loops can maintain an image in short-term memory, and how long term learning and recognition can occur. The bio-engineering principle thus is clear.

Terminology

Damasio has the following use of terms:

- Mind consists of mental events, conscious or nonconscious.
- A baby has a mind but still no ego and hence no consciousness (and only nonconsciousness, like sleepwalking, or a state like mankind must have been during much of evolution).
- When "self comes to mind" then the child grows aware and gets conscious.
- A person can sleep and not be aware (in the sense of awake) but still have a mind and a self, for example in dreaming (e.g. as shown from dreams that are remembered).

My suggestion is not to adopt this terminology. In my terminology – that in my perception is rather common – mind and consciousness are other words for the same. The mind is awake **and** aware **and** equipped with a sense of self. Thus mind = consciousness.

My **preferred usage of terms** remains with Merriam-Webster (2011). In particular for **mind** I confer with explanation 2a and 2b (while 2c is Damasio's choice but problematic): '2a: the element or complex of elements in an individual that feels, perceives, thinks, wills, and especially reasons; 2b: the conscious mental events and capabilities in an organism; 2c: the organized conscious and nonconscious adaptive mental activity of an organism'.

As Damasio clarifies, the whole brain is required to create consciousness. Mind / consciousness only works because there is a large apparatus working at the nonconscious level. For example, the very experience that I think about a circle and handle it, can only happen because of the combined effort of all brain parts. This cannot happen at the nonconscious level since there is no supporting apparatus for it. At the nonconscious level there are only brain events – and not that concept of that circle.

A mental act of thinking about a circle, i.e. the thought of a circle, only 'exists' when awake and aware (which rather is the self). My preference is that thinking about a circle (e.g. as a Platonic ideal circle) can be classified as a 'mental event'. The nonconscious brain may contain a map of a circle at the level of reality1, but it only becomes the mental object of a circle when it enters the mind and becomes part of reality2. It is not a convincing suggestion that reality2 can also exist 'nonconsciously', as if all these mental objects are also available 'nonconsciously'. Reality1 would contain the elements that can be combined to create reality2. (And it does not seem proper to assume that there is a reality3 consisting of 'reality2 turned nonconscious'.)

It would be a (bad) figure of speech to say 'I was nonconsciously thinking about a circle when suddenly the Pythagorean Theorem popped up in my mind.' Properly seen, nonconscious thought is different from conscious thought, and the associated 'circles' would not be the same.

Saying that a baby is unconscious tends to mean that it is not awake, for example due to a car crash that knocked it unconscious. It is only because of brain research that we now employ the term 'nonconscious', and we may use it to mean that a baby has no developed self. The change of language game is a bit awkward / surprising. However, all this should not force us to change our basic vocabulary. I would rather say that a baby has an 'less developed mind' and 'low level of consciousness' and 'less developed self'. Damasio's terminology 'self comes to mind' then would rather be 'the child develops a mind = grows conscious' (by passing particular tests on levels of consciousness).

It is surprising that Damasio in his effort to reach a wider public departs from standard English and attaches different meanings to “mind” and “consciousness” than the standard vocabulary gives us – and adopts the confusing use of Merriam-Webster entry 2c. Perhaps it is the advantage of simple mathematical concepts like line and circle that help us to cement that standard vocabulary as right and proper, and convince Damasio that he should adapt his terms in new editions.

Consciousness as an experience

What standard science cannot explain is the experience of consciousness itself, nor, importantly, sensations of pleasure and pain as they occur in the mind. Inanimate atoms bump around without pleasure and pain, how can it be that e.g. pain feels so horrible in my mind ? It is not sufficient to point to pleasure and pain processes in the brain. What is at issue is the mental experience.

The standard approach is to accept that the universe has human beings who say that they are conscious. In reductionist mode, consciousness is seen as an emergent property of unconscious particles and waves. This is adequate with respect to reality0 but it inadequately denies the mental experience in reality2.

While we are accustomed to recognize the five senses of touch, tasting (touch by molecular form), smelling (touch by molecular form), hearing (movement of air) and seeing (touch by light), and while we are increasingly including other aspects such as balance or sense of temperature or magnetism, it might be a suggestion to see the mind as another ‘sense’. The mind is a ‘sense’ in that it observes something special from reality0 that is not observed by the other senses. One might argue that physical ‘matter’ and ‘electromagnetic waves’ create the brain and the brain creates the mind, ergo: the mind must be physical too. Mankind has tended to reject this view. As a conscious mind myself I follow that tendency since for me too it **seems** that there is something quite un-physical about my conscious mind. Let us be exact: it is un-physical2standard in terms of standard physical models that we design in reality2, but it will still be physical0 in terms of reality0 since the latter is our definition of all that is.

For me that ‘seems’ is so strong that I would rather say **appears**. While the brain is physical1 and apparently consists of atoms that bump into each other without pleasure and pain, the mind is non-physical in standard physics, i.e. un-physical2standard. The brain creates the situation that a conscious human being has that experience of consciousness – with pleasure and pain. Thus arises the hypothesis that a certain constellation of atoms might be sensitive to another dimension that is not directly accessible by the normal five senses (forms of touching) that deal in commonly understood manner with reality0.

The mental experience, including pleasure and pain, regarded as a ‘sense’, need not be more complex than the normal five senses. It could already exist in single cells, as Damasio explains that those already show reactions that remind of reward and punishment systems. Clearly, we already can make robots with light sensors that ‘look for light’ to charge their batteries, so that feed and flight mechanisms don’t require such a sixth sense. If a cell has a storage of energy it can ‘reward’ itself by its release. Inanimate feed and flight however do not explain the mental experience, neither in the aggregate. The existence of that robot model does not mean that single cell organisms work like that.

If we agree that reality0 (our definition of all that there is) has a dimension that relates to mental experience, such as pleasure and pain, that are non-physical2standard, then single cell organisms

might use a simpler feed and flight system by directly plugging into pleasure and pain sensations that occur in that mental dimension. Subsequently, the brain builds up from such single cell mechanisms to copy the similar process at the macro level, as feed and flight are evolutionary successful at that level too. There are the two models that the brain either collects and manages myriad sensations of pleasure and pain or mainly magnifies those of some core cells. It remains to be seen whether this hypothesis generates testable propositions or only remains a logical possibility that serves to perhaps satisfy a curious mind.

The above seems just a reformulation of the old suggestion of a Platonic world where ideal concepts might 'exist', and even a human soul as part of a deity. It is not quite the same. The common ground is the acceptance that consciousness does not fit physical2standard models and thus is non-physical2standard. The difference is the intention to work with testable hypotheses.

In a discussion on brain research and mathematics education it is useful to have this paragraph on the Platonic world. It is an obvious question "where does this ideal circle exist ?" and apparantly many mathematicians tend to have a Platonic conception about this. The answer suggested here is that abstraction does not imply 'existence' in terms of reality0. Consciousness would however be part of reality0. Our universe has human beings who say that they are conscious. The "Cogito ergo sum" is a valid argument of an intellect observing existence. Pleasure and pain are such that there are no good alternatives but to accept reality0 too. The (creation of the) thought of a circle in a conscious mind does not imply anything other than what happens similarly when a circle is drawn (approximated) on a piece of paper: it is not eternal, like the standard Platonic world would suppose.

PM. Smith, in Descartes (1958), observes that Augustinus already formulated the "Cogito ergo sum" argument but not compactly. For various years I myself adopted the idea that consciousness was rather an illusion in an entirely physical world, yet, an illusion presupposes some intellect that suffers it, which is the "cogito ergo sum" argument. We can observe a dog running after a ball and presume that it has all kinds of illusions, and obviously it is not conscious as in recognizing a mirror image of itself, yet we should not let ourselves be carried away by *sorites* types of paradoxes.

Mind – body equivalence

As Damasio uses 'feeling' for the mind and 'emotion' for the body. I have been tempted to suggest to use Anglo-Saxon English for the mind and Latin-French English for the body. (Shakespeare mixes these uses too, for both the common people and the high court.) But I am afraid that this would not do. It is better to label with 1 and 2, or (our model of the) brain and mind. Thus we have brain and mind, and the question arises how these are linked.

Damasio (2010) forwards the hypothesis – and the Note in his appendix p314-317 is surprisingly the clearest on this: 'that mental states and brain states are essentially equivalent.'

A key paragraph is Damasio (2010:316): 'Accepting the hypothesized mental/neural equivalence is especially helpful with the vexing problem of downward causality. Mental states do exert their influence on behavior, as can easily be revealed by all manner of actions executed by the nervous system and the muscles at its command. The problem, some will say the mystery, has to do with how a phenomenon that is regarded as non-physical – the mind – can exert its influence on the very physical nervous system that moves us to action. Once mental states and neural states are

regarded as the two faces of the same process, one more Janus out to trick us, downward causality is less of a problem.'

I agree with this. At the same time however he emphasizes that the whole brain is required to generate consciousness. Is this still consistent with that equivalence ? If I study the same proof on the Pythagoren Theorem either sitting in a train or in a chair at home, I suppose that my mental states would be the same but brain elements would be different since in the first case they have to deal with noise and wobbling of the train. Perhaps the hypothesis is saved by the qualifier 'essential'.

I would rather use 'association' instead of 'equivalence'. Logically it would seem strange that the same brain state might generate different mind states (for where would the random element come from ?). Conversely different brain states might generate the same mind state. (We would have to rely on the person who is experimented with whether it indeed is the same mind state.)

Fully adopting 'equivalence' might have the awkward consequence that physical reality0 deterministically generates mental states. I don't mind such models when they enhance clarity but I reject the idea that they are more than models – since what is the proof ?

Subsequently, with respect to that key paragraph, I would like to suggest an analogy for the relation between the mental experience and the process of organisation of brain activity.

An analogy is gravity. In the Newtonian model a shift of my hand has an instant impact on the Moon. Possibly at the Big Bang not all dimensions have exploded and there still is an instanteneous connection between all matter. Another model might require gravity waves that travel at the speed of light. A deterministic model also explains the move of the hand, assumes a common cause and then a simultaneous development such that both my hand moves and the Moon is affected. With the latter analogy we may model states of brain and mind, without getting lost in causality.

Thus what remains is that brain mechanisms evolve to ever better organisation, and that the latter is experienced in the mind. It is not that the mind 'causes' organisation but the ever better organisation at the brain level carries the flag of ever better mental states. As long as we are aware of the proper causal order, figures of speech that employ downward causality might help to increase understanding how mental concepts affect actions.

This usefulness is increased by the social environment of the human species. To my taste Damasio does not mention it sufficiently often (neither in the development of the mirror neurons) that mankind developed in a social situation. If we do include it with more emphasis then we see that the flux of social events is quite 'unphysical' as the mind (except for awareness). For example, non-physical minds understand what happens when a mother embraces her child. Physics shows us two masses huddling together, and nothing more. There is no need to suggest that the non-physical affects the physical, since there is a physical apparatus as a common cause. Yet it is true that my conscious understanding of these events includes mental notions. And organised minds communicate in a social fabric, that adds another layer of organisation namely in all kinds of cultural dimensions. Importantly, when notions are encoded in language, the signs and sounds of language impact on the mind (of others), and evolution, which means that the 'unphysical' organisation finds a physical counterpart.

A note on the flash of insight

Colignatus (2011b) COTP mentions that it can be a good teaching technique to create a conundrum, then drop the punchline, and then all students 'get it' in 'a flash'. I happened to see the project on humor by gymnasium students Riksen, Wenink and Wellink (2011), and their reference called my attention to Helmuth Plessner (1970); and this also gave Agnes Heller (2005) and her student Stuart Grant (2008) and Ulrike Günther (2002). I have not studied these references but am struck by Grant's formulation of one of Heller's propositions: 'Laughter is the instinct of reason'. Clearly there is a cognitive element in humor – it must be about something. Clearly, teaching and learning can employ the 'get it in a flash' that also occurs in a joke. It might be, however, that we may define humor as incongruity and that incongruity is the natural state of the mind, and that instead of humor it is serious reasoning that is the exception that needs an explanation. For example, if $1 + 1 = 2$ is serious, and $1 + 1 = \sqrt{4}$ parody, and $1 + 1 = 1 + 2 – 1$ sarcasm, and $1 + 1 = 1$ is irony (by teasingly using + for *), then the serious item only remains after elimination of all incongruity. It is hard work to cut through the forest of inconsistencies and inefficiencies to find something that works seriously. Neurons in a brain branch out with some randomness and the mechanism to align them into something that works might be that 'flash'. With a feedback to consciousness that records with pleasure that something has been learned. I just leave this further with this note.

Consciousness and free will

Brain research literature suggests that the brain first starts an action and that the conscious mind records it only with a delay of 0.2 milliseconds. Damasio also discusses the Dijksterhuis experiment on the quality of conscious decision making. It must be observed though that, again, the whole brain is required to construct the mind. Thus it does not make much sense to say that volition arises only after the fact. The sense of free will is already in the whole process of the generation of the state of mind. It still is the whole brain, that comprises the individual, that generates both the act and its conscious state. It is not proper to make another distinction.

It is another thing to observe that a person can react in instinct or in automation and only later correct this. Someone can train his nonconscious mind to react on impulse so that the conscious mind has only the results to deal with. This is great for math exams. These are practical issues on the brain / mind interaction and not essential questions on volition versus determinism.

Proper versus 'realistic' mathematics

Traditional mathematics relies much on abstraction. Euclid's *Elements* is didactically dubious as a book, but Euclidean geometry with elements such as point, line, triangle and circle might not be too abtract and may be grasped at the different Van Hiele levels, working from the intuitions towards the deductive stage. After the Sputnik launch, America started the New Math education program, that however was didactically destructive. It tells us much about mathematics teachers (of that period ?) that they tried it (collectively), before finally there was a revolt by parents and a few very disgusted mathematicians. In Holland, Freudenthal proposed an alternative both to the traditional programme and the New Math. He adopted notions of applied mathematics to introduce concepts close to the life experience of students and his method was called "realistic mathematics education" (RME). Van den Heuvel-Panhuizen (1998) explained 10 years ago that the project was 30 years old and still "work in progress". She wishes to clarify that RME stands for "making something real in your mind". It tells us much about mathematics teachers (of that period ?) that they tried it (collectively), before finally there was a revolt by parents and a few

very disgusted mathematicians. Elementary schools have recently been instructed to work harder on arithmetic, highschools meet with higher requirements of traditional skills, and teachers have managed to introduce more abstract math for the talented.

The didactic point made in EWS and COTP is that the Dutch RME programme, even adapted in the current manner, still is didactically misguided. It does not properly value that mathematics is abstract by nature. The real life contexts can distract the student. RME says that it uses real contexts to work towards abstraction, but in truth it creates confusion. It does not properly distill the abstract notions from those real contexts and the student is left to discover the abstractions himself. Freudenthal says that he wants "guided re-invention" to help the student toward the abstractions but doesn't deliver on the promise. My suggestion is to focus on the notion that thought is abstract by nature, so that events in reality0 have already made a transformation when they are processed in reality2 even at the lowest Van Hiele level. As Van Hiele explains, the jump from one level to the next one is made by classifying, sorting, ordering and abstracting on what has been achieved at that lower level. The student can be given activities that imply this kind of processing and once the ground is fertile the required abstract notions can be given (with that flash of insight).

The true problem in mathematics education is rather that mathematicians are trained for theory while when they meet students in class then these are real life students, which requires an empirical mind set. They try to solve their problems by resorting to tradition in what they teach, but this causes mathematical concepts and notations that are quite cumbersome. EWS advises national parliaments to investigate their national education in mathematics, and COTP provides a primer for mathematical education and an existence proof how it could be done.

Once these awkward confusions are resolved, a reason to link up mathematics education and brain research is that mathematical concepts are abstract and provide mental events, as the mind can be defined as the flux of mental events. Looking at both angles at the same time may give results.

EWS for example refers to Gladwell (2008:228): "we store digits in a memory loop that runs for about two seconds". English numbers are cumbersome to store. Gladwell then quotes Stanislas Dehaene: "(…) the prize for efficacy goes to the Cantonese dialect of Chinese, whose brevity grants residents of Hong Kong a rocketing memory span of about 10 digits." Apparently fractions in Chinese are clearer too. Instead of two-fifths it would use two-from-five. First creating fifths indeed is an additional operation. Perhaps the West is too prim on the distinction between the ratio 2:5 and the number 2/5. Perhaps it does really not make a difference except in terms of pure theory – the verb of considering the ratio and the noun of the result (called "number" when primly formalized in a number theory). In evidence based education we can imagine experiments testing the various variants, with some support in brain scanning. There is a great variety in children and how they learn but perhaps that variety is a bit less when asked very specific operations under a scan.

Conclusions

The subject matter is brain research and the education in mathematics. Small comments in this present paper are the following.

(1) This paper forwards a sharper distinction than Damasio (2010) between reality0, 1 and 2. Damasio of course uses these aspects. Reality2 is nothing but how our mind models reality0. But

by explicitly naming them we enhance clarity. For example, continuity might exist in reality0 but still has to be created for reality2. The Van Hiele levels of mathematical understanding are levels in reality2 itself (per mathematical concept). Each level could have different degrees of Gardner's multiple intelligences (http://www.howardgardner.com/FAQ/faq.htm). We better understand the language games and the multiple meanings of the 'circle', if we would want brain research to assist us in teaching and learning about the circle.

(2) Damasio's mind-body equivalence only seems to survive because of his use of the qualifier 'essential'. It is better to use the term 'association' so that various brain states may associate with the same mental state e.g. as strictly defined by the thought about a mathematical concept. It makes sense to refer to mental states in discussion of the organisation of brain activity. For mathematics education it is important to create well-defined concepts, as a useful definition of mental states is that they contain well-defined concepts.

(3) This discussion would provide a better base for the use of abstract notions in the education of mathematics. Mathematical concepts like line and circle are abstract notions (at some Van Hiele level) and made of the same stuff as the mind itself. Education can use this to its advantage. The approach by Freudenthal with 'realistic math' seems to cause too much distraction from the intended abstraction.

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